Engineering Analytic Principles and Predictive Computational Skills for K-12 Students:

Statistics on High School Age-Possible Statics Topics for Engineering and Technology Educators and Curriculum Developers

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In the most recent decade, middle and high schools across the United States have tried to incorporate engineering design into traditional technology curriculum, with various degrees of success; however, "the fragmented focus and lack of a clear curriculum framework" had been "detrimental to the potential of the field and have hindered efforts aimed at achieving the stated goals of technological literacy for all students" (Smith and Wicklein, 2007, pp. 2-3). A report issued on September 8, 2009, by the Committee on K-12 Engineering Education established by the National Academy of Engineering and the National Research Council, titled Engineering in K-12 Education: Understanding the Status and Improving the Prospects (2009), confirmed the existence of similar problems, such as the "absence of a clear description of which engineering knowledge, skills, and habits of mind are most important, how they relate to and build on one another, and how and when (i.e., at what age) they should be introduced to students" (pp. 7-8; p. 151). K-12 engineering curriculum in the United States remains skeletal so far; its main focus is on generic design process using a "trial-and-error" approach; and the coverage of analytic and predictive knowledge contents is generally in an "ad hoc" fashion and not sequentially structured. In response to the above problems, many scholars have voiced their points of view. Hacker (2011) pointed out that "trial-and-error problem solving takes substantial classroom time, and often does not allow teachers and students to focus on the most important learning goals." Lewis (2007, pp. 846-848) discussed the need to: (a). establish a "codified body of knowledge that can be ordered and articulated across the grades" instead of short term efforts focused on a particular topic or unit, and (b). make engineering education a coherent system with the creation of content standards for the subject area, in line with science and technology education.

High School Age-possible Engineering Topics (Statics)

Research Questions and Practical Conceptual Framework

The above evaluation of the current status of K-12 engineering education in the United States could lead to these questions: (1). "How could we determine what engineering analytic principles and predictive skills from what subject should be taught to students at what Grade in the K-12 curriculum, in a rational and scientific way?" (2). "How could we make sure that what students learned from high school engineering curriculum could be transferred to university programs?" Based on the way engineering curriculum has been historically developed, I have constructed a practical conceptual framework to answer the above two questions. If we read any typical information sheet for university level undergraduate engineering program, we will see that the courses are organized in a sequence based on the fulfillment of pre-requisites in mathematics, physics, chemistry, technology and previous engineering courses; and these pre-requisites are usually listed in course descriptions. Therefore, we could hypothesize that the same principles used historically in the development of curricular structure in university undergraduate engineering programs could apply to the selection of K-12 age-possible engineering analytic principles and predictive skills for any particular Grade, and for any particular subject of engineering. In addition, based on the fact that university undergraduate engineering textbooks, especially those used in foundation courses (such as statics, dynamics, strength of materials, engineering economics, etc.), all contain portions that are based on pre-calculus mathematics and scientific principles which are usually covered in K-12 mathematics and science courses, we could also hypothesize that these precalculus portions of engineering topics could possibly be taught at various Grade levels, provided that the pre-requisite pre-calculus mathematics and science principles have been covered in previous Grade levels (or in some cases, taught as special topics); and the coverage of such pre-requisites are usually mandated by the performance standards in mathematics and science established by any particular state. This conceptual framework has been used as a practical tool for the initial determination of 9th grade age-possible statics and fluid mechanics topics. The step-by-step procedure or the "ideal" procedure (Locke, 2009a, pp. 26-27) includes the following (Figure *I*): (1) selection of data source (selection of popular university undergraduate engineering textbooks and other instructional and learning materials); (2) analysis of data source (careful reading of every paragraph in the body text as well as relevant computational formulas to find and record the pre-requisite mathematics skills and scientific principles needed for each topic; (3) comparison (between the recorded mathematics and science pre-requisites, and my interpretation of the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state, in this case, the State of Georgia, to determine the Grade level for the agepossible inclusion of the topics). I selected the State of Georgia's Standards as a reference for the research because (1) the University of Georgia, my alma mater, gave me the opportunity to study the subject of K-12 engineering education and (2) many professors at the College of Education and the College of Agricultural and Environmental Sciences (Department of Biological and Agricultural Engineering) offered me valuable advice and criticism. Due to the fact that the variations among the K-12 mathematics and science performance standards of the 50 states are not substantial, the outcomes of the research should apply to other states with some reasonable adaptations.



Figure 1. The original research data table used to initially determine high school 9th Grade age-possible statics topics.

After completing all lower-division undergraduate mechanical engineering courses plus two upper-division ones, and conducting a careful and fairly thorough examination of many other college-level engineering textbooks, I have made the conclusion that all engineering textbooks include the following major elements:

- (1) **Descriptive and informational:** Paragraphs, data tables, charts, graphs, illustrations and photos that explain natural phenomena, scientific principles, properties of materials, behaviors of structures and systems, in "plain English," without going into the details of analytic and predictive computations using formulas based on mathematics skills.
- (2) <u>Analytic and predictive:</u> Mathematics-based formulas, including those used in pre-requisite physics and chemistry concepts, principles and analysis, and those used in engineering analysis and design, and step-by-step procedures, including sample problems with solutions, for analyzing problems, predicting outcomes, or designing systems or products; and these mathematics

skills could be at either pre-calculus level, i.e., arithmetic, trigonometry, geometry, algebra, or at calculus level, i.e., integration and differentiation.

In terms of the relative amount of each of the above major elements in the overall composition of the content of the textbooks, all sets of college-level engineering textbooks used in any particular course or subject could be classified into three major categories; i.e., (1) Mixture of Pre-calculus and Calculus, (2) Heavily Pre-calculus, and (3) Heavily Descriptive and Informational. It takes different amounts of time and efforts to examine different sets of textbooks under different categories in order to tentatively determine and select K12 agepossible engineering content knowledge and skills, including descriptive and informational materials, analytic and predictive computational formulas and step-by-step problem solving procedures; and the procedure of this examination include (a) interpretation of the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state, in this case, the State of Georgia, to create a coded list of items of mathematics, physics and chemistry concepts and skills, such as $M4G3 \rightarrow 4^{th} Grade$ (1B) shown in Figure 1, from the original online government document, to be used for comparison with the mathematics, physics and chemistry concepts and skills found from the relevant textbooks; (b) paragraph-by-paragraph or page-by-page examination of the selected textbooks for the extraction and documentation of the mathematics, physics and chemistry concepts and skills needed to understand the content and to solve homework problems; and (c) comparison between the interpreted, itemized and coded lists of Performance Standards and the items extracted from the textbooks, to tentatively determine and select sections and chapters in the textbooks that could be K12 age-possible. In the United States, we have a very decentralized management structure for the publication and adaptation of textbooks; and this is especially true for textbooks used in the institutions of higher education where professors usually select textbooks out of their own choices free from government intervention; for any college courses or subject, we can find several excellent and popular textbooks, all of them cover a majority of similar topics plus a small number of different ones; therefore, to be holistic and comprehensive, at least two of the most popular textbooks will be used, one as the "primary source of data" and the rest as "secondary source of data" and "additional sources of data." The nature of composition of the above-mentioned three major categories of textbooks and the average amount of time it takes for their examination are as follows:

1. <u>Mixture of Pre-calculus and Calculus:</u> Textbooks under this category include, for the undergraduate mechanical engineering major, those used in the courses of statics, dynamics, strength of materials, electric machines, mechanical design, aerodynamics, fluid mechanics, electrical circuits, heat transfer, thermodynamics, and others. For these textbooks, calculus and pre-calculus skills are used intermittently throughout substantial portions of most of the chapters. These textbooks are usually voluminous and the numbers of pages range from 600 to 900. Therefore, a thorough investigation of all paragraphs, formulas, and even sample problems in the textbooks, and a very detailed record of all pertinent information in tabular forms is needed to determine and to select K-12 age-possible engineering topics for different grade levels. My research projects on the subjects of statics and fluid mechanics have been completed this way. This procedure is very thorough and time-consuming and for one subject, it takes

between 3 to 5 weeks for one textbook (the "primary source of data"), and additional 1 to 2 weeks for another textbook (the "secondary source of data" used to pick up additional K12 age-possible topics); these amounts of time cover careful reading of all chapters, sections, and even appendices and glossaries throughout the entire textbooks, analysis and recording of mathematics, physics and chemistry concepts and skills involved, typing of titles of chapters, sections, formulas, names of pre-requisite items, write-ups of conclusions, as well as a section-by-section review. Typing of titles of chapters, sections, and formulas could take up to one third of the above amounts of time needed for the research. It is the exact or "ideal" procedure advocated in my published Vision Paper.

- 2. Heavily Pre-calculus: Textbooks under this category include those used in the courses of engineering economics, probability and statistics, and others. For these textbooks, the mathematics skills involved in the majority or even the overwhelming majority of chapters and sections are at pre-calculus level; the calculus skills involved in a few sections or chapters are the very beginning ones such as [first integral] and [first derivative]; and the principles and skills related to physics and chemistry are also the very basic ones; therefore, a less time-consuming approach is used to determine and select K12 age-possible engineering topics, by carefully examine each page in the textbooks to record (1) the pre-calculus level mathematics skills as well as physics and chemistry concepts, principles and skills found in all pages; (2) the calculus-level mathematics skills found in some pages, the page numbers where these calculus skills are found, the numbers and names as well as the pages ranges of the sections involving the calculus skills; and (3) result of comparison between the pre-calculus skills as well as physics and chemistry concepts and skills found throughout the textbooks, and the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state, in this case, the State of Georgia, to determine the earliest Grade level for the agepossible inclusion of the topics. My research projects on the subjects of engineering economics, probability and statistics, and engineering materials have been completed this way. This procedure is fairly thorough but much less time-consuming because no record of mathematics-based formulas or typing of the names of chapters and sections of the textbooks that involve only precalculus mathematics skills is needed, and for one subject, it takes between 5 to 7 days for one textbook (the "primary source of data") and additional 2 to 4 days for another textbook (the "secondary source of data"). These amounts of time cover careful reading of all chapters, sections, and even appendices and glossaries throughout the entire textbooks, analysis and recording of mathematics, physics and chemistry concepts and skills involved as pre-requisites, typing of page numbers and titles of chapters and sections involving calculus stills as well as numbers of the individual pages involved, write-ups of conclusions, as well as a section-by-section review. It is a convenient and "ad hoc" revision of the "ideal" procedure advocated in my published Vision Paper.
- 3. <u>Heavily Descriptive and Informational:</u> Textbooks under this category include those used in the courses of introduction to science, engineering and technology, ethics and professionalism in engineering, and others. These textbooks involve little or no

mathematics skills; their primary goal is to expose students to broad knowledge about engineering, science, technology, as well as their relationship with "other stuff" such as society, ecology, legal system, philosophy, and others. Similar method as the one used for the "Heavily Pre-calculus" textbooks is used here but the amounts of time spent is substantially reduced because, for the "Heavily Descriptive and Informational" textbook, mathematics, physics and chemistry pre-requisites are rarely involved. For one subject, it takes between 1 to 3 days for one textbook (the "primary source of data") and additional 1 to 2 days for another textbook (the "secondary source of data"). These amounts of time cover careful reading of all chapters, sections, and even appendices and glossaries throughout the entire textbooks, analysis and recording of a few mathematics, physics and chemistry concepts and skills involved as pre-requisites, write-ups of conclusions, as well as a section-by-section review. It is a convenient, "ad hoc" and more drastic revision of the "ideal" procedure advocated in my published Vision Paper.

Sources of Data

Table 1 lists (1) the college-level Textbook 1 (the "primary source of data") used for the extraction of analytic and predictive principles and computational formulas related to the subject of statics, and (2) the instructor's or student's solution manuals used to double-check for the mathematics computational skills needed for the study of various topics of statics contained in the selected.

	Main Textbook	Instructor's So	lution Manuals
Title	Vector Mechanics for Engineers	Instructor's and Solutions Manual to	Instructor's and Solutions Manual to
	Statics, 7 th Edition	Accompany Vector Mechanics for	Accompany Vector Mechanics for
		Engineers – Statics, 7 th Edition,	Engineers – Statics, 7 th Edition,
		Volume 1	Volume 2
Authors	Ferdinand P. Beer & E. Russell	Ferdinand P. Beer & E. Russell	Ferdinand P. Beer & E. Russell
	Johnston & Elliot R. Eisenberg	Johnston & Elliot R. Eisenberg	Johnston & Elliot R. Eisenberg
Publisher	McGraw-Hill Higher Education	McGraw-Hill Higher Education	McGraw-Hill Higher Education
Year	2004	2004	2004
ISBN	0-07-230493-6	10: 0072536055	10: 0072962623

Table 1. Data Source (Statics Textbook)

Original Research Data

Table 2 (Statics Topic List - Engineering Topics Mathematics and Science Pre-requisite Completion Chart for the Subject of Statics) constitute the original research data; the way data is recorded and analyzed is shown in *Figure 1*. The leftmost column in the Table 1 (Statics Topic List - Engineering Topics Mathematics and Science Pre-requisite Completion Chart for the Subject of Statics) contains

- 1. The titles of each section under a particular chapter in the selected textbook, which in general represent particular sets of statics related engineering analytic and predictive principles, in a qualitative and explanatory way;
- 2. Computational formulas, which symbolically represent the above engineering analytic and predictive principles, in a quantitative and mathematical way.

The middle column (divided into two sub-columns, i.e., Math, and Physics/Chemistry, is reserved for recording mathematics and science (physics and chemistry) pre-requisites for understanding the knowledge content and using the formulas contained in the particular chapters or sections, as well as the grade levels these pre-requisites are expected to be fulfilled according to Georgia Performance Standards.

The right column records the prediction on the possible grade a particular chapter or section could be taught to K12 students.

Findings from the Research Project

Table 3A (Pre-Calculus Based Statics Topics That Possibly Could Be Taught at 9th Grade), Table 3B (Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portion of Statics Topics to 9th Grade Students), Table 4A (Calculus Based Statics Topics for Post-Secondary Engineering Education), and Table 4B (Pre-Requisite Math and Science Topics to Be Reviewed Before Teaching the Calculus Portion of Statics Topics) constitute the outcomes of the research, which is presented here to the public as "public domain" knowledge, to be used as sources of reference by K12 engineering and technology education practitioners and curriculum developers, nationally and internationally, including myself, of course!

Table 2. Statics Topics ListEngineering Topics Mathematics and Science Pre-requisite Completion Chart for the Subject of Statics

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.) Moth Physics		Pos Gi to S the	sible ade Start Topic
	Math	Physics	Sec	Ch
Chapter 1: Introduction				
1.1: What Is Mechanics?	[coordinate system] (M4G3) \rightarrow 4 th (2B)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	9 th
1.2: Fundamental Concepts and Principles	[measurement: time] (M2M2) $\rightarrow 2^{nd}$ (2C)	[Newton's 1 st , 2 nd and 3 rd Laws] (SP1) \rightarrow 9 th (3C)		
\vec{F} \vec{a} \vec{a} \vec{a} \vec{a} $\vec{m}_1 \vec{m}_2$	[Parallelogram Law for the Addition of	[acceleration] (S8P3) \rightarrow 8th (3C)		
$a = - \Longrightarrow F = ma$ $F_{AB} = -F_{BA}$ $F = G - \frac{1}{r^2}$	Force/Vector Graphics] (MA3A10) \rightarrow 11 th (2H)	[Newton's Law of Gravitation] (S8P5) \rightarrow 8 th (3C)		
	\rightarrow To be taught as a special math topic	[scientific inquiry] (S7CS9) \rightarrow 7 th (3B)		
1.3: Systems of Units	[unit conversion] (M6M1) \rightarrow 6 th (2C)	N/A	6 th	
1.4: Conversion from One System of Units to Another				
1.5: Method of Problem Solution	[problem-solving] (M3N5) \rightarrow 3 rd (2A)	N/A	3 rd	1
1.6: Numerical Accuracy	$[\text{percent}] (M5N5) \rightarrow 5^{\text{th}} (2A)$	N/A	5 th	1
Chapter 2: Statics of Particles				
2.1: Introduction	[four operations] (M1N3) \rightarrow 1 st (2A) +	[force] (S4P3) \rightarrow 4 th (3A)	4 th	9 th
Forces in a Plane	$(M2N3) \rightarrow 2^{nd} (1A), \text{ or } (M7N1) \rightarrow 7^{th} (2A)$			
2.2: Force on a Particle. Resultant of Two Forces	[coordinate system] (M4G3) \rightarrow 4 th (2B)			
2.3: Vectors	[vector graphics] (MA3A10) \rightarrow 9 th (2H)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	1
2.4: Addition of Vectors	\rightarrow To be taught as a special math topic			
2.5: Resultant of Several Concurrent Forces				
2.6: Resolution of a Force into Components	[vector graphics] (MA3A10) \rightarrow 9 th (2H)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	1
2.7: Rectangular Components of a Force. Unit Vectors	[trigonometric functions] (MA2G2) \rightarrow 9 th (2F)			
2.8: Addition of Forces by Summing x and y Components	[four operations] (M1N3) \rightarrow 1 st (2A) +	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	8 th	1
$\vec{F} = F_r \hat{i} + F_r \hat{j} F_r = F \cos \theta$	$(M2N3) \rightarrow 2^{nd} (2A), \text{ or } (M7N1) \rightarrow 7^{th} (2A)$		1	
F F	[square root] (M8N1) \rightarrow 8 th (2A)			
$F_y = F \sin \theta \tan \theta = \frac{Y}{F} F = \sqrt{F_x^2 + F_y^2}$	[trigonometric functions] (MA2G2) \rightarrow 10 th (2F)			
1 x	\rightarrow To be taught as a special math topic		1	
	[coordinate system] (M4G3) \rightarrow 4 th (2B)		1	1
2.9: Equilibrium of a Particle	[sigma notation] (M6N1) \rightarrow 6 th (1A) or (MA1A3)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (2C)	9 th	7
$R = \sum F = F_1 + F_2 + \dots = 0 \Rightarrow R_x = \sum F_x = 0 R_y = \sum F_y = 0$	\rightarrow 9 th (2E) \rightarrow To be taught as a special math topic			
$R_z = \sum F_z = 0$	[four operations] (M1N3) \rightarrow 1 st (2A) +		1	1
	$(M2N3) \rightarrow 2^{nd} (1A)$, or $(M7N1) \rightarrow 7^{th} (2A)$			

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to S the 2	sible ade Start Topic
	Math	Physics	Sec	Ch
Chapter 2: Statics of Particles (Continued)				
2.10: Newton's First Law of Motion	[four operations] (M1N3) \rightarrow 1 st (2A) +	[Newton's 1 st , 2 nd and 3 rd Laws] (SP1) \rightarrow 9 th (3C)	9 th	9 th
2.11: Problems Involving the Equilibrium of a Particle.	$(M2N3) \rightarrow 2^{nd} (1A), \text{ or } (M7N1) \rightarrow 7^{th} (2A)$	[acceleration] (S8P3) \rightarrow 8th (3C)		
Free-Body Diagrams			oth	4
Porces in Space 2.12: Rectangular Components of a Force in Space $F_y = F \cos \theta_y$, $F_h = F \sin \theta_y$ $F_x = F_h \cos \phi = F \sin \theta_y \cos \phi$, $F_z = F_h \sin \phi = F \sin \theta_y \sin \phi$ $F^2 = F_y + F_h = F_y + F_x + F_z$, $\rightarrow F = \sqrt{F_x + F_y + F_z}$ $F_x = F \cos \theta_x$, $F_y = F \cos \theta_y$, $F_z = F \cos \theta_z$, $(0^\circ < \theta_{x,y,z} < 180^\circ)$ $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$ $\vec{F} = F(\cos \theta_x \hat{i} + \cos \theta_y \hat{j} + \cos \theta_z \hat{k})$ $\cos \theta_x = \frac{F_x}{F} = \frac{d_x}{d} = \frac{R_x}{R}$, $\cos \theta_y = \frac{F_y}{F} = \frac{d_y}{d} = \frac{R_y}{R}$, $\cos \theta_z = \frac{F_z}{F} = \frac{d_z}{d} = \frac{R_z}{R}$ $\theta_{x(y,z)} = \cos^{-1} \frac{F_{x(y,z)}}{F} = \cos^{-1} \frac{d_{x(y,z)}}{d}$, $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ $\hat{\lambda} = \cos \theta_x \hat{i} + \cos \theta_y \hat{j} + \cos \theta_z \hat{k}$, $\hat{\lambda} = \frac{\vec{F}}{F}$, $\hat{i} = \frac{d_x}{d}$, $\hat{j} = \frac{d_y}{d}$, $\hat{k} = \frac{d_z}{d}$ $\cos^2 \theta_y + \cos^2 \theta_y + \cos^2 \theta_z = 1$, $\rightarrow \hat{\lambda}_x^2 + \hat{\lambda}_x^2 + \hat{\lambda}_z^2 = 1$	<pre>[10th operations] (M1N3) → 1⁻⁴ (2A) + (M2N3) → 2nd (1A), or (M7N1) → 7th (2A) [square root] (M8N1) → 8th (2A) [trigonometric functions] (MA2G2) → 10th (2F) → To be taught as a special math topic [coordinate system] (M4G3) → 4th (2B)</pre>	[10rce] (34r3) 7 4" (3A) or (38r3) 7 8" (3C)	9	

Engineering Subject: Statics Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to S the '	sible ade Start Fopic
	Math	Physics	Sec	Ch
Chapter 2: Statics of Particles (Continued)				
2.13: Force Defined by Its Magnitude and Two Points on Its Line of Action $\overline{MN} = d_x \hat{i} + d_y \hat{j} + d_z \hat{k}$ $\hat{\lambda} = \frac{\overline{MN}}{MN} = \frac{1}{d} \left(d_x \hat{i} + d_y \hat{j} + d_z \hat{k} \right)$ $d_{x(y,z)} = x(y,z)_2 - x(y,z)_1 d = \sqrt{d_x^2 + d_y^2 + d_z^2}$ $\overline{F} = F\hat{\lambda} = \frac{F}{d} \left(d_x^2 \hat{i} + d_y^2 \hat{j} + d_z^2 \hat{k} \right)$ $F_x = \frac{Fd_x}{d} F_y = \frac{Fd_y}{d} F_z = \frac{Fd_z}{d}$	[four operations] (M1N3) \rightarrow 1 st (2A) + (M2N3) \rightarrow 2 nd (1A), or (M7N1) \rightarrow 7 th (2A) [square root] (M8N1) \rightarrow 8 th (1A) [trigonometric functions] (MA2G2) \rightarrow 10 th (2F) \rightarrow To be taught as a special math topic [coordinate system] (M4G3) \rightarrow 4 th (2B)	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's 1 st , 2 nd and 3 rd Laws] $(SP1) \rightarrow 9^{th} (3C)$	9 th	9 th
2.14: Addition of Concurrent Forces in Space $\vec{R} = \sum \vec{F} R = \sqrt{R_x^2 + R_y^2 + R_z^2}$ $R_x \hat{i} + R_y \hat{j} + R_z \hat{k} = (\sum F_x)\hat{i} + (\sum F_y)\hat{j} + (\sum F_z)\hat{k}$	[four operations] (M1N3) \rightarrow 1 st (2A) + (M2N3) \rightarrow 2 nd (1A), or (M7N1) \rightarrow 7 th (2A) [square root] (M8N1) \rightarrow 8 th (2A) [trigonometric functions] (MA2G2) \rightarrow 10 th (2F) \rightarrow To be taught as a special math topic [coordinate system] (M4G3) \rightarrow 4 th (2B)	[force] $(S4P3) \rightarrow 4^{\text{th}} (3A)$ or $(S8P3) \rightarrow 8^{\text{th}} (3C)$ [Newton's 1 st , 2 nd and 3 rd Laws] $(SP1) \rightarrow 9^{\text{th}} (3C)$	9 th	
2.15: Equilibrium of a Particle in Space $R = \sum F = F_1 + F_2 + = 0 \rightarrow R_x = \sum F_x = 0 R_y = \sum F_y = 0 R_z = \sum F_z = 0$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $R_x = \sum F_x = 0 aF_1 + bF_2 + cF_3 = 0$ $R_y = \sum F_y = 0 dF_1 + eF_2 + fF_3 = 0$ $R_z = \sum F_z = 0 gF_1 + hF_2 + iF_3 = 0$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} aF_1 + bF_2 + cF_3 \\ aF_1 + eF_2 + fF_3 \\ gF_1 + hF_2 + iF_3 \end{bmatrix}$	[coordinate system] (M4G3) \rightarrow 4 th (2B) [four operations] (M1N3) \rightarrow 1 st (2A) + (M2N3) \rightarrow 2 nd (1A), or (M7N1) \rightarrow 7 th (2A) [linear algebra](MA2A6) (MA2A7) (MA2A8) (MA2A9) \rightarrow 10 th (2G) \rightarrow To be taught as a special math topic	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's 1 st , 2 nd and 3 rd Laws] $(SP1) \rightarrow 9^{th} (3C)$	9 th	

Engineering Subject. States Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gi to S	ssible rade Start
		-	the '	Topic
	Math	Physics	Sec	Ch
Chapter 3: Rigid Bodies - Equivalent Systems	s of Forces			
3.1: Introduction	[four operations] (M1N3) \rightarrow 1 st (2A) +	$[\text{force}] (\text{S4P3}) \rightarrow 4^{\text{th}} (\text{3A})$	6 th	9 th
3.2: External and Internal Forces	$(M2N3) \rightarrow 2^{nu} (2A)$	$[motion] (SKP2) \rightarrow K (3A)$		
3.3: Principle of Transmissibility. Equivalent Forces	[geometry: point, axis/line, 3D body] (M6G1) (M6G2) (M6M3) \rightarrow 6 th (2B)			
3.4: Vector Product of Two Vectors	[trigonometric functions] (MA2G2) \rightarrow 10 th (2F)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	-
$\vec{V} = \vec{P} \times \vec{Q}$ $V = PQ \sin \theta$ $\vec{V} \perp \vec{P}$ $\vec{V} \perp \vec{Q}$ $\vec{V} \perp Plane_{\vec{P},\vec{Q}}$	 → To be taught as a special math topic [cross product] → To be taught as a special math topic 	[motion] (SKP2) → K (3A)		
$\vec{P} \times (\vec{O}_1 + \vec{O}_2) = \vec{P} \times \vec{O}_1 + \vec{P} \times O_2 (\vec{P} \times \vec{O}) \times \vec{S} \neq \vec{P} \times (\vec{O} \times \vec{S})$				
$\vec{V} = \vec{Q} \times \vec{P} = -(\vec{P} \times \vec{Q}) \vec{Q} \times \vec{P} \neq \vec{P} \times \vec{Q} \vec{P} \times \vec{Q} = -\vec{V}$				
$\vec{P} \times \left(\vec{Q}_1 + Q_2\right) = \vec{P} \times \vec{Q}_1 + \vec{P} \times Q_2 \vec{Q} \times \vec{P} \neq \vec{P} \times \vec{Q}$				
$\vec{V} = \vec{Q} \times \vec{P} = -(\vec{P} \times \vec{Q}) \vec{P} \times \vec{Q} = -\vec{V} \vec{V} = \vec{P} \times \vec{Q} (\vec{P} \times \vec{Q}) \times \vec{S} \neq \vec{P} \times (\vec{Q} \times \vec{S})$				
3.5: Vector Products Expressed in Terms of Rectangular	[trigonometric functions] (MA2G2) \rightarrow 10 th (2F)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	
Components	[cross product] \rightarrow To be taught as a special math			
$i \times i = j \times j = k \times k = 0$ $i \times j = k$ $j \times k = i$ $k \times i = j$	$[dot product] \rightarrow To be taught as a special math$			
$\hat{i} \times \hat{k} = -\hat{j}$ $\hat{j} \times \hat{i} = -\hat{k}$ $\hat{k} \times \hat{j} = -\hat{i}$	topic			
$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k} \vec{Q} = Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}$	topic			
$\vec{V} = \vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$				
$V_x = P_y Q_z - P_z Q_y V_y = -(P_x Q_z - P_z Q_x) = P_z Q_x - P_x Q_z$				
$V_z = P_x Q_y - P_y Q_x$				

Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to S the 2	sible ade Start Topic
	Math	Physics	Sec	Ch
Chapter 3: Rigid Bodies - Equivalent System	s of Forces (Continued)			
3.6: Moment of a Force about a Point	[four operations]	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	9 th
$\vec{M}_0 = \vec{r} \times \vec{F}$ $M_0 = rF\sin\theta = Fd$	$(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$			
$\vec{r} = \vec{v}^{0 \to A}$ $\theta = \measuredangle - d + \vec{F}$	[geometry: point, axis/line, 3D body]			
$V = V_{position}$ $V = Z_{\vec{r} \rightarrow \vec{F}}$ $u \neq 1$	$(M6G1) (M6G2) (M6M3) \rightarrow 6^{th} (2B)$			
i j k	[\cross product] \rightarrow To be taught as a special math			
$\vec{M}_0 = \vec{r} \times \vec{F} = \begin{vmatrix} x & y & z \end{vmatrix} = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$	topic			
$F_x = F_y = F_z$	$[dot product] \rightarrow 1o be taught as a special math$			
M = vF = zF $M = -(vF = zF) = zF = vF$ $M = vF = vF$	$\frac{\text{topic}}{(144246)(144247)}$			
$M_x - yr_z - zr_y M_y = -(\lambda r_z - zr_x) - zr_x - \lambda r_z M_z - \lambda r_y - yr_x$	$(MA2A8) (MA2A0) \rightarrow 10^{\text{th}} (2\text{C})$			
	\rightarrow To be taught as a special math tonic			
37: Varignon's Theorem	[four operations]	[force] (S4P3) $\rightarrow 4^{\text{th}}$ (3A) or (S8P3) $\rightarrow 8^{\text{th}}$ (3C)	Qth	-
$\vec{\pi} \cdots (\vec{E} + \vec{E} + \vec{n}) = \vec{\pi} \cdots \vec{E} + \vec{\pi} \cdots \vec{E}$	$(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$		-	
$r \times (F_1 + F_2 +) = r \times F_1 + r \times F_2 +$	[cross product] \rightarrow To be taught as a special math			
	topic			
	[dot product] \rightarrow To be taught as a special math			
	topic			
3.8: Rectangular Components of the Moment of a Force	[four operations]	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	
$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$	$(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$			
	[cross product] \rightarrow To be taught as a special math			
$M_B = r_{A/B} \times F = \begin{vmatrix} x_{A/B} & y_{A/B} \\ z_{A/B} \end{vmatrix}$	topic			
F_{x} F_{y} F_{z}				
			1	
$\vec{r}_{A/B} = x_{A/B}\hat{i} + y_{A/B}\hat{j} + z_{A/B}\hat{k}$				
$\mathbf{r} = \mathbf{r} - \mathbf{r} \mathbf{v} = \mathbf{v} - \mathbf{v} \mathbf{z} = \mathbf{z} = \mathbf{z}$				

Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Complet	ion Grade (Georgia Performance Standard Code) iples] (GPS Code) → Grade (Table No.)	Pos Gi to S the	ssible rade Start Topio
	Math	Physics	Sec	Ch
Chapter 3: Rigid Bodies - Equivalent System	ns of Forces (Continued)			
3.9: Scalar Product of Two Vectors $\vec{P} \bullet \vec{Q} = PQ \cos \theta = P_x Q_x + P_y Q_y + P_z Q_z \theta = \angle_{\vec{P} \to \vec{Q}}$ $\vec{P} \bullet \vec{Q} = \vec{Q} \bullet \vec{P} = \vec{P} \bullet (\vec{Q} + \vec{Q}) = \vec{P} \bullet \vec{Q} + \vec{P} \bullet \vec{Q}$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [dot product] \rightarrow To be taught as a special math	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	9 th
$P_{OL} = \vec{P} \bullet \hat{\lambda} = P_x \cos \theta_x + P_y \cos \theta_y + P_z \cos \theta_z$ (More formulas on p. pp. 94-95)	topic			
3.10: Mixed Triple Product of Three Vectors $\vec{S} \bullet (\vec{P} \times \vec{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [cross product] \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	
3.11: Moment of a Force about a Given Axis $M_{OL} = \hat{\lambda} \bullet \vec{M}_{O} = \hat{\lambda} \bullet (\vec{r} \times \vec{F}) = \begin{vmatrix} \lambda_{x} & \lambda_{y} & \lambda_{z} \\ x & y & z \\ F_{x} & F_{y} & F_{z} \end{vmatrix}$ (More formulas on p. pp. 98)	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [dot product] \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	
3.12: Moment of a Couple $\vec{M} = \vec{r} \times \vec{F}$ $M = rF \sin \theta = Fd$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [trigonometric functions] $(MA2G2) \rightarrow 10^{th} (2F)$ \rightarrow To be taught as a special math topic [cross product] \rightarrow To be taught as a special math topic	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [motion] $(SKP2) \rightarrow K (3A)$ [lever] $(S4P3) \rightarrow 4^{th} (3A)$	9 th	
3.13: Equivalent Couples $F_1d_1 = F_2d_2$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [geometry: point, axis/line, 3D body] $(M6G1) (M6G2) (M6M3) \rightarrow 6^{th} (2B)$	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [motion] $(SKP2) \rightarrow K (3A)$ [lever] $(S4P3) \rightarrow 4^{th} (3A)$	6 th	

Engineering Analytic Topics & Typical Formulas	Engineering Analytic Topics & Typical Formulas [Pre-requisite Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Possible Grade to Start the Topi	
	Math	Physics	Sec	Ch
Chapter 3: Rigid Bodies - Equivalent System	s of Forces (Continued)			
3.14: Addition of Couples $\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 \vec{M} = \vec{M}_1 + \vec{M}_2$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [cross product] \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	9 th
3.15: Couples Can Be Represented by Vectors	[vector graphics] (MA3A10) \rightarrow 11 th (2H) \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	
3.16: Resolution of a Given Force Into a Force at <i>O</i> and a Couple $\vec{M}_{O'} = \vec{r}' \times \vec{F} = (\vec{r} + \vec{s}) \times \vec{F} = \vec{r} \times \vec{F} + \vec{s} \times \vec{F} \vec{M}_{O'} = \vec{M}_O + \vec{s} \times \vec{F}$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [cross product] \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	-
3.17: Reduction of a System of Forces to One Force and One Couple $\vec{R} = \sum \vec{F} \vec{M}_{O}^{R} = \sum \vec{M}_{O} = \sum (\vec{r} \times \vec{F})$ $\vec{M}_{O'}^{R} = \vec{M}_{O} + \vec{s} \times \vec{R} \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $\vec{F} = F_{x}\hat{i} + F_{y}\hat{j} + F_{z}\hat{k} \vec{R} = R_{x}\hat{i} + R_{y}\hat{j} + R_{z}\hat{k}$ $\vec{M}_{O}^{R} = M_{x}^{R}\hat{i} + M_{y}^{R}\hat{j} + M_{z}^{R}\hat{k}$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [cross product] \rightarrow To be taught as a special math topic	[force] $(S4P3) \rightarrow 4^{\text{th}} (3A)$ or $(S8P3) \rightarrow 8^{\text{th}} (3C)$	9 th	-
3.18: Equivalent Systems of Forces $\sum \vec{F} = \sum \vec{F'} & \& \sum \vec{M}_0 = \sum \vec{M'}_0$ $\sum \vec{F} = \sum \vec{F'} & and \sum \vec{M}_0 = \vec{M}_0'$ $\sum F_x = \sum F'_x \sum F_y = \sum F'_y \sum F_z = \sum F'_z$ $\sum M_x = \sum M'_x \sum M_y = \sum M'_y \sum M_z = \sum M'_z$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [coordinate system] $(M4G3) \rightarrow 4^{th} (2B)$	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	8 th	
3.19: Equipollent Systems of Vectors	[vector graphics] (MA3A10) \rightarrow 11 th (2H) \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	
3.20: Further Reduction of a System of Forces	[coordinate system] (M4G3) \rightarrow 4 th (2B)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	8 th	1

Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.) Math Physics		Pos Gr to S the 7	sible ade Start Topic
	Math	Physics	Sec	Ch
Chapter 3: Rigid Bodies - Equivalent Systems	s of Forces (Continued)			
3.21: Reduction of a System of Forces to a Wrench $p = \frac{M_1}{R} M_1 = \frac{\vec{R} \cdot \vec{M}_O^R}{R} p = \frac{M_1}{R} = \frac{\vec{R} \cdot \vec{M}_O^R}{R^2}$ $\vec{M}_1 = p\vec{R} \rightarrow \frac{\vec{M}_1 + \vec{r} \times \vec{R} = \vec{M}_O^R}{p\vec{R} + \vec{r} \times \vec{R} = \vec{M}_O^R}$	[four operations] $(M1N3) \rightarrow 1^{st} (1A) + (M2N3) \rightarrow 2^{nd} (2A)$ [geometry: point, axis/line, 3D body] $(M6G1) (M6G2) (M6M3) \rightarrow 6^{th} (2B)$ [trigonometric functions] $(MA2G2) \rightarrow 10^{th} (2F)$ \rightarrow To be taught as a special math topic [dot product] \rightarrow To be taught as a special math topic [cross product] \rightarrow To be taught as a special math topic	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [motion] $(SKP2) \rightarrow K (2A)$ [lever] $(S4P3) \rightarrow 4^{th} (2A)$	9 th	9 th
Chapter 4: Equilibrium of Rigid Bodies	•			
4.1: Introduction	[sigma notation] (M6N1) \rightarrow 6 th (1A) or (MA1A3)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	9 th
$\sum \vec{F} = 0 \sum F_x = 0 \sum F_y = 0 \sum F_z = 0$	\rightarrow 9 th (2E) \rightarrow To be taught as a special math topic [coordinate system] (M4G3) \rightarrow 4 th (2B)	[Newton's 3^{rd} Law: Action and Reaction]		
$\sum \vec{M}_o = \sum (\vec{r} \times \vec{F}) = 0 \sum M_x = 0 \sum M_y = 0 \sum M_z = 0$	[coordinate system] (N405) 7 4 (2D)	(311) 77 (30)		
4.2: Free-Body Diagram	-			
Equilibrium in Two Dimensions 4.3: Reactions at Supports and Connections for a Two- Dimensional Structure				
4.4: Equilibrium of a Rigid Body in Two Dimensions				
$F_z = 0 M_x = M_y = 0 M_z = M_o$				
$\sum F_x = 0 \sum F_y = 0 \sum M_o = 0$				
$\sum M_A = 0 \sum M_B = 0 \sum M_C = 0$				
4.5: Statically Indeterminate Reactions. Partial Constraints]			
4.6: Equilibrium of a Two-Force Body				
4.7: Equilibrium of a Three-Force Body				1

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.) Math		Pos Gr to S the T	sible ade Start Fopic
	Math	Physics	Sec	Ch
Chapter 4: Equilibrium of Rigid Bodies (Con	tinued)			1
Equilibrium in Three Dimensions	[sigma notation] (M6N1) \rightarrow 6 th (1A) or (MA1A3)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	9 th
4.8: Equilibrium of a Rigid Body in Three Dimensions	\rightarrow 9 th (2E) \rightarrow To be taught as a special math topic	[Newton's 3 rd Law: Action and Reaction]		
$\sum F = 0 \sum M_o = \sum (r \times F) = 0$	[coordinate system] (M403) 7 4 (2B)	$(SP1) \rightarrow 9^{\text{an}}(3C)$		
$\sum F_x = 0 \sum F_y = 0 \sum F_z = 0$				
$\sum M_x = 0 \sum M_y = 0 \sum M_z = 0$				
4.9: Reactions at Supports and Connections for a Three-				
Dimensional Structure				
Chapter 5: Distributed Forces: Centroids and	Centers of Gravity			
5.1: Introduction	[coordinate system] (M4G3) \rightarrow 4 th (2B)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	PS
Areas and Lines	[sigma notation] (M6N1) $\rightarrow 6^{\text{th}}$ (1A) or (MA1A3)	[Newton's Law of Gravitation] (S8P5) $\rightarrow 8^{\text{m}}$ (3C)		
5.2: Center of Gravity of a Two-Dimensional Body $P(a_{1}, \sum E_{1}, W = AW + AW + AW$	$\rightarrow 9^{\text{m}}$ (2E) $\rightarrow 10$ be taught as a special main topic [integration] $\rightarrow 12^{\text{th}}$ (To be taught)			
$\mathbf{F}_{uue}: \sum \mathbf{F}_{z}: \mathbf{W} = \Delta \mathbf{W}_{1} + \Delta \mathbf{W}_{2} + \dots + \Delta \mathbf{W}_{n}$				
$\sum M_{y}: \overline{x}W = x_{1}\Delta W + x_{2}\Delta W + \dots + x_{n}\Delta W$				
$\sum M_x: \overline{y}W = y_1 \Delta W + y_2 \Delta W + \dots + y_n \Delta W$				
$W = \int dW \overline{x}W = \int xdW \overline{y}W = \int ydW$				
Wire: $\sum M_y$: $\overline{x}W = \sum x\Delta W$ $\sum M_x$: $\overline{y}W = \sum y\Delta W$				
5.3: Centroids of Areas and Lines	[measurement: area, weight, thickness]	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	
Plate: $\Delta W = \gamma \Delta A$ $W = \gamma A$ $\overline{x}A = \int x dA$ $\overline{y}A = \int y dA$	$(M6M1) (M6M2) \rightarrow 6^{th} (2C)$ [integration] $\rightarrow 12^{th}$ (To be taught)	[Newton's Law of Gravitation] (S8P5) $\rightarrow 8^{\text{th}}$ (3C)		
<i>Line</i> : $\Delta W = \gamma a \Delta L$ $\bar{x}L = \int x dL$ $\bar{y}L = \int y dL$				

Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Complet	ion Grade (Georgia Performance Standard Code)	Pos	sible
gg,,	[Pre-requisite Math Skills/Science Princ	ciples] (GPS Code) → Grade (Table No.)	Gi to S the	'ade Start Topi
	Math	Physics	Sec	Ch
Chapter 5: Distributed Forces: Centroids an	nd Centers of Gravity (Continued)			
5 4. First Momenta of Arross and Lines	[integration] → 12 th (To be taught) [coordinate system] (M4G3) → 4 th (2B) [two-dimensional figures: circle, arc, triangle, ellipse, parabolic]	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's Law of Gravitation] $(S8P5) \rightarrow 8^{th} (3C)$	PS	PS
$\bar{x}A = Q_y = \int xdA$ $\bar{y}A = Q_x = \int ydA$	 (M1G1) (M1G2) → 1st (1B) + (MA2G4) → 10th (2F) → To be taught as a special math topic [special two-dimensional figures: parabolic spandrel, general spandrel] → To be taught as a special math topic 			
5.5: Composite Plates and Wires $\overline{X} \sum W = \sum \overline{x}W \overline{Y} \sum W = \sum \overline{y}W$ $Q_y = \overline{X} \sum A = \sum \overline{x}A Q_x = \overline{Y} \sum A = \sum \overline{y}A$	[coordinate system] (M4G3) \rightarrow 4 th (2B) [sigma notation] (M6N1) \rightarrow 6 th (1A) or (MA1A3) \rightarrow 9 th (2E) \rightarrow To be taught as a special math topic [measurement: area, weight, thickness] (M6M1) (M6M2) \rightarrow 6 th (2C)	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's Law of Gravitation] $(S8P5) \rightarrow 8^{th} (3C)$	PS	
5.6: Determination of Centroids by Integration $Q_y = \bar{x}A = \int \bar{x}_{el} dA Q_x = \bar{y}A = \int \bar{y}_{el} dA$	[integration] $\rightarrow 12^{\text{th}}$ (To be taught) [coordinate system] (M4G3) $\rightarrow 4^{\text{th}}$ (2B) [areas of geometric shapes: circle, triangle, etc.] (M5M1) $\rightarrow 5^{\text{th}}$ and (2B) (M6M2) $\rightarrow 6^{\text{th}}$ (2C)	[force] $(S4P3) \rightarrow 4^{\text{th}} (3A)$ or $(S8P3) \rightarrow 8^{\text{th}} (3C)$ [Newton's Law of Gravitation] $(S8P5) \rightarrow 8^{\text{th}} (3C)$	PS	
5.7: Theorems of Pappus-Guldinus $A = 2\pi \bar{y}L$ $V = 2\pi \bar{y}A$	[integration: area of surface of revolution, curve, volume of body of revolution] → 12 th (To be taught)	[force] $(S4P3) \rightarrow 4^{\text{th}} (3A)$ or $(S8P3) \rightarrow 8^{\text{th}} (3C)$ [Newton's Law of Gravitation] $(S8P5) \rightarrow 8^{\text{th}} (3C)$	PS	
5.8: Distributed Loads on Beams $W = \int_{O}^{L} w dx W = \int dA = A (OP)W = \int x dW (OP)A = \int_{O}^{L} x dA$	[coordinate system] (M4G3) \rightarrow 4 th (2B) [integration] \rightarrow 12 th (To be taught) [areas of geometric shapes: circle, triangle, etc.] (M5M1) \rightarrow 5 th and (2B) (M6M2) \rightarrow 6 th (2C)	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's Law of Gravitation] $(S8P5) \rightarrow 8^{th} (3C)$	PS	
5.9: Forces on Submerged Surfaces $w = bp = b\gamma h$	[areas of geometric shapes: circle, triangle, etc.] (M5M1) \rightarrow 5 th and (2B) (M6M2) \rightarrow 6 th (2C)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	8^{th} \rightarrow PS	1

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.) Math Physics		Pos Gr to S the '	sible :ade Start Topic
	Math	Physics	Sec	Ch
Chapter 5: Distributed Forces: Centroids an	d Centers of Gravity (Continued)			
Volumes5.10: Center of Gravity of a Three- Dimensional Body.Centroid of a Volume $\bar{x}W = \int xdW$ $\bar{y}W = \int ydW$ $\bar{z}W = \int zdW$ $\bar{x}V = \int xdV$ $\bar{y}V = \int ydV$ $\bar{z}V = \int zdV$	[coordinate system] (M4G3) \rightarrow 4 th (2B) [volume: sphere, cone, pyramid] (M5M4) \rightarrow 5 th (1B) (M6M3) \rightarrow 6 th (2B) (MA1G5) \rightarrow 9 th (2F) [volume: ellipsoid, paraboloid] \rightarrow To be taught as a special math topic	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's Law of Gravitation] $(S8P5) \rightarrow 8^{th} (3C)$	PS	PS
5.11: Composite Bodies $\overline{X} \sum W = \sum \overline{x}W \overline{Y} \sum W = \sum \overline{y}W \overline{Z} \sum W = \sum \overline{z}W$ $\overline{X} \sum V = \sum \overline{x}V \overline{Y} \sum V = \sum \overline{y}V \overline{Z} \sum V = \sum \overline{z}V$	[integration] → 12 th (To be taught) [integration: area of surface of revolution, curve, volume of body of revolution] → 12 th (To be taught)			
5.12: Determination of Centroids of Volumes by				
Integration				
$\overline{x}V = \int \overline{x}_{el} dV \overline{y}V = \int \overline{y}_{el} dV \overline{z}V = \int \overline{z}_{el} dV \overline{x}V = \int \overline{x}_{el} dV$				
Chapter 6: Analysis of Structures				
6.1: Introduction	[sigma notation] (M6N1) \rightarrow 6 th (1A) or (MA1A3)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	9 th
Trusses	\rightarrow 9 th (2E) \rightarrow To be taught as a special math topic	[Newton's 3rd Law: Action and Reaction]		
6.2: Definition of a Truss	[four operations] (M1N3) \rightarrow 1 st (1A) + (M2N3) \rightarrow	$(SP1) \rightarrow 9^{th} (3C)$		
6.3: Simple Trusses	2^{nd} (1A), or (M7N1) \rightarrow 7 th (2A)			
6.4: Analysis of Trusses by the Method of Joints	[coordinate system] (M4G3) \rightarrow 4 ^m (2B)			
6.5: Joints under Special Loading Conditions				
6.6: Space Trusses				
6.7: Analysis of Trusses by the Method of Sections				
6.8: Trusses Made of Several Simple Trusses				
<u>Frames and Machines</u> 6.9: Structures Containing Multiforce Members	[trigonometric functions] (MA2G2) \rightarrow 10 th (2F) [coordinate system] (M4G3) \rightarrow 4 th (2B) [sigma notation] (M6N1) \rightarrow 6 th (1A) or (MA1A3)	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's 3 rd Law: Action and Reaction] $(SP1) \rightarrow 9^{th} (3C)$	9 th	
6.10: Analysis of a Frame	\rightarrow 9 th (2E) \rightarrow To be taught as a special math topic			
6.11: Frames Which Cease to Be Rigid When Detached	[four operations] (M1N3) \rightarrow 1 st (2A) +			
from Their Supports	$(M2N3) \rightarrow 2^{nd} (1A), \text{ or } (M7N1) \rightarrow 7^{th} (2A)$			

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to S the 2	sible ade Start Fopic
	Math	Physics	Sec	Ch
Chapter 6: Analysis of Structures (Continued)			
6.12: Machines	[sigma notation] (M6N1) → 6 th (1A) or (MA1A3) → 9 th (2E) → To be taught as a special math topic [four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (1A), or (M7N1) → 7 th (2A) [trigonometric functions] (MA2G2) → 10 th (2F) → To be taught as a special math topic [coordinate system] (M4G3) → 4 th (2B)	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [Newton's 3 rd Law: Action and Reaction] $(SP1) \rightarrow 9^{th} (3C)$	9 th	9 th
Chapter 7: Forces in Beams and Cables			1	1
7.1: Introduction 7.2: Internal Forces in Members	[sigma notation] (M6N1) $\rightarrow 6^{\text{th}}$ (1A) or (MA1A3) $\rightarrow 9^{\text{th}}$ (2E) \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	PS
	[four operations] (M1N3) \rightarrow 1 st (2A) +			
Beams	$(M2N3) \rightarrow 2^{nd} (2A), or (M7N1) \rightarrow 7^{th} (2A)$			
7.3: Various Types of Loading and Support	[integration] \rightarrow 12 th (To be taught)			
7.4: Shear and Bending Moment in a Beam				
7.5: Shear and Bending-Moment Diagrams		$\mathbf{f}_{0} = 1_{0} (0 + \mathbf{D}_{0}) \mathbf{N}_{0} \mathbf{f}_{0} (0 + \mathbf{N}_{0}) \mathbf{N}_{0} \mathbf{f}_{0} (0 - \mathbf{N}_{0}) \mathbf{N}_{0} \mathbf{f}_{0} \mathbf{f}_$	DC	_
7.6: Relations among Load, Shear, and Bending Moment $\frac{dV}{dx} = -w$ $V_D - V_C = -\int_{x_C}^{x_D} w dx = -wx = = -(\text{Area under load curve between C an D})$ $\frac{dM}{dx} = V$	[sigma notation] (M6N1) → 6 th (1A) or (MA1A3) → 9 th (2E) → To be taught as a special math topic [four operations] (M1N3) → 1 st (1A) + (M2N3) → 2 nd (1A), or (M7N1) → 7 th (2A) [integration] → 12 th (to be taught) [differentiation] → 12 th (to be taught)	[force] $(54P3) \rightarrow 4^{\text{tr}} (3A)$ or $(58P3) \rightarrow 8^{\text{tr}} (3C)$	PS	
$M_D - M_C = \int_{x_C}^{\infty} V dx = -(\text{Area under shear curve between C an D})$				

Engineering Subject: Statics Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Poss Gra to Sta To	ible ade rt the pic
	Math	Physics	Sec	Ch
Chapter 7: Forces in Beams and Cables (Con	ntinued)			
Cables 7.7: Cables with Concentrated Loads	[sigma notation] (M6N1) \rightarrow 6 th (1A) or (MA1A3) \rightarrow 9 th (2E) \rightarrow To be taught as a special math topic	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	8 th	PS
7.8: Cables with Distributed Loads $T\cos\theta = T_o$ $T\sin\theta = W$ $T = \sqrt{T_o^2 + W^2}$ $\tan\theta = \frac{W}{T_o}$	[trigonometric functions] (MA2G2) \rightarrow 10 th (2F) \rightarrow To be taught as a special math topic [four operations] (M1N3) \rightarrow 1 st (2A) + (M2N2) \rightarrow 2 nd (2A) or (M7N1) \rightarrow 7 th (2A)			
7.9: Parabolic Cable $y = \frac{wx^2}{2T_o}$	[square root] (M8N1) \rightarrow 8 th (2A)			
7.10: Catenary $T = \sqrt{T_o^2 + w^2 s^2} c = \frac{T_o}{w} T_o = wc W = ws T = w\sqrt{c^2 + s^2}$ $dx = ds \cos\theta = \frac{T_o}{T} ds = \frac{wcds}{w\sqrt{c^2 + s^2}}$ $x = \int_o^s \frac{ds}{\sqrt{1 + \frac{s^2}{c^2}}} = c \left[\sinh^{-1} \frac{s}{c} \right]_o^s = c \sinh^{-1} \frac{s}{c}$	[sigma notation] (M6N1) → 6 th (1A) or (MA1A3) → 9 th (2E) → To be taught as a special math topic [trigonometric functions] (MA2G2) → 9 th (2F) [four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (2A), or (M7N1) → 7 th (2A) [square root] (M8N1) → 8 th (2A) [integration] → 12 th (to be taught) [differentiation] → 12 th (to be taught)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	
$s = c \sinh \frac{x}{c} y = c \cosh \frac{x}{c} y^2 - s^2 = c^2 T_o = wc W = ws$ $T = wy h = y_A = c$				

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to Sta To	sible ade art the ppic
	Math	Physics	Sec	Ch
Chapter 8: Friction				
8.1: Introduction	[four operations] (M1N3) \rightarrow 1 st (2A) +	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	9 th	PS
8.2: The Laws of Dry Friction. Coefficients of Friction	$(M2N3) \rightarrow 2^{nd} (2A), \text{ or } (M7N1) \rightarrow 7^{th} (2A)$			
$F_m = \mu_s N$ $F_k = \mu_k N$	[trigonometric functions] (MA2G2) \rightarrow 10 th (2F)			
8.3: Angles of Friction	\rightarrow To be taught as a special math topic			
$ \tan \phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N} \longrightarrow \tan \phi_s = \mu_s $	[surface] (M6M4) $\rightarrow 6^{\text{un}}$ (2B)			
$ \tan \phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N} \longrightarrow \tan \phi_k = \mu_k $				
8.4: Problems Involving Dry Friction				
8.5: Wedges				
8.6: Square-Threaded Screws				
$Q = P \frac{a}{r} L = nP$				
8.7: Journal Bearings. Axle Friction				
$M = Rr\sin\phi_k \approx Rr\mu_k r_f = r\sin\phi_k \approx r\mu_k$				
8.8: Thrust Bearings. Disk Friction	[four operations] (M1N3) \rightarrow 1 st (2A) +	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	
$\Delta M = r\Delta F = \frac{r\mu_k P\Delta A}{\pi \left(R_2^2 - R_1^2\right)}$	$(M2N3) \rightarrow 2^{nd} (2A), \text{ or } (M7N1) \rightarrow 7^{th} (2A)$ [integration] $\rightarrow 12^{th}$ (to be taught)			
$M = \frac{\mu_k P}{\pi (R_2^2 - R_1^2)} \int_0^{2\pi} \int_{R_1}^{R_2} r^2 dr d\theta = \frac{\mu_k P}{\pi (R_2^2 - R_1^2)} \int_0^{2\pi} \left[\frac{r^{2+1}}{2+1} \right]_{R_1}^{R_2} d\theta$				
$=\frac{\mu_k P}{\pi (R_2^2 - R_1^2)} \int_0^{2\pi} \frac{1}{3} (R_2^3 - R_1^3) d\theta$				
Ring area : $M = \frac{2}{3} \mu_k P \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$ Full circle : $M = \frac{2}{3} \mu_k P R$				
8.9: Wheel Friction. Rolling Resistance	[four operations] (M1N3) \rightarrow 1 st (2A) +	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	8 th	
Pr = Wb	$(M2N3) \rightarrow 2^{nd} (2A), \text{ or } (M7N1) \rightarrow 7^{th} (2A)$			

Engineering Subject: Statics	Engineering Subject: Statics						
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)		Poss Gra	sible ade			
	[Pre-requisite Math Skills/Science Princ	iples] (GPS Code) → Grade (Table No.)	to Sta To	rt the pic			
	Math	Physics	Sec	Ch			
Chapter 8: Friction (Continued)							
8.10: Belt Friction $\ln \frac{T_2}{T_1} = \mu_s \beta \frac{T_2}{T_1} = e^{\mu_s \beta}$ (For other formulas, refer to pp. 451-452)	[sigma notation] (M6N1) → 6 th (1A) or (MA1A3) → 9 th (2E) → To be taught as a special math topic [four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (2A), or (M7N1) → 7 th (2A) [trigonometric functions] (MA2G2) → 10 th (2F) → To be taught as a special math topic [logarithmic functions] (MA2A4) → 10 th (2E) → To be taught as a special math topic [integration] → 12 th (to be taught) [differentiation] → 12 th (to be taught)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	PS			

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Poss Gra to Star Toj	ible ide rt the pic
	Math	Physics	Sec	Ch
Chapter 9: Distributed Forces: Moments of Iner	rtia		1	
9.1: Introduction	[integration] $\rightarrow 12^{\text{m}}$ (to be taught)	[force] (S4P3) $\rightarrow 4^{\text{in}}$ (3A) or (S8P3) $\rightarrow 8^{\text{in}}$ (3C)	PS	PS
Moments of Inertia of Areas	$[differentiation] \rightarrow 12^{th} (to be taught)$	$[power] (SP3) \rightarrow 9^{\text{th}} (3C)$		
9.2: Second Moment, or Moment of Inertia, of an Area	[four operations] (M1N3) \rightarrow 1 st (2A) + (M2N2) \rightarrow 2nd (2A) or (M7N1) \rightarrow 7th (2A)			
$R = \int kydA = k \int ydA M = \int ky^2 dA = k \int y^2 dA$	$(M2N3) \rightarrow 2^{m}(2A), \text{ of } (M/N1) \rightarrow 7^{m}(2A)$			
$R = \int \gamma y dA = \gamma \int y dA M_x = \int y^2 dA = \gamma \int y^2 dA$	[area] (MSM3) (MSM4) \rightarrow 5 th (2B) [square root] (M8N1) \rightarrow 8 th (2A)			
9.3: Determination of the Moment of Inertia of an Area by	[coordinate system] (M4G3) \rightarrow 4 th (2B)			
Integration	[areas of geometric shapes: circle, triangle]			
$I_x = \int y^2 dA I_y = \int x^2 dA dA = b dy dI_x = y^2 b dy$	$(MSM1) \rightarrow 5^{\text{tri}} (2B)$ [geometric shapes: ellipse] (MA2G4) $\rightarrow 10^{\text{th}}$			
$I_{x} = \int_{O}^{h} by^{2} dy = \frac{1}{3}BH^{3} dI_{x} = \frac{1}{3}y^{3} dx dI_{y} = x^{2} dA = x^{2} y dx$	$(2F) \rightarrow$ To be taught as a special math topic [three-dimensional bodies: thin rectangular			
9.4: Polar Moment of Inertia	plate, rectangular prism] (M5M4) \rightarrow 5 th (2B)			
$J_{o} = \int r^{2} dA = \int (x^{2} + y^{2}) dA = \int y^{2} dA + \int x^{2} dA$	[three-dimensional bodies: slender rod, circular cylinder, cone] (M6M3) \rightarrow 6 th (2B)			
$J_o == I_x + I_y$	[three-dimensional bodies: circular cone,			
9.5: Radius of Gyration of an Area	sphere]			
$I_x = k_x^2 A \rightarrow k_x = \sqrt{\frac{I_x}{A}} I_y = k_y^2 A \rightarrow k_y = \sqrt{\frac{I_y}{A}}$	$(M2G2) \rightarrow 2^{nd} (2B)$ [trigonometric functions] $(MA2G2) \rightarrow 10^{th} (2F)$ \rightarrow To be taught as a special math topic			
$J_o = k_o^2 A \rightarrow k_o = \sqrt{\frac{J_o}{A}}$	$[cross product] \rightarrow To be taught as a special math topic$			
9.6: Parallel-Axis Theorem	[gradient: "del"] \rightarrow 12 th (to be taught)			
$I = \int y^2 dA$	[linear algebra] (MA2A6) (MA2A7) (MA2A8)			
$I = \int y^{2} dA = \int (y' + d)^{2} dA = \int y'^{2} dA + 2d \int y' dA + d^{2} \int dA$	(MA2A9) → 10 ^m (2G)			
$I = \bar{I} + Ad^{2} k^{2} = \bar{k}^{2} + d^{2} J_{o} = \bar{J}_{o} + Ad^{2} or k_{o}^{2} = \bar{k}_{o}^{2} + d^{2}$				
			1	

Engineering Analytic Topics & Typical Formulas Chapter 9: Distributed Forces: Moments of Iner 9.7: Moments of Inertia of Composite Areas The second	Math & Science Pre-requisite Topics & Completi [Pre-requisite Math Skills/Science Princt] Math	on Grade (Georgia Performance Standard Code) iples] (GPS Code) → Grade (Table No.) Physics	Poss Gra to S the T	sible ade tart Copic
Chapter 9: Distributed Forces: Moments of Iner 9.7: Moments of Inertia of Composite Areas	Math	Physics	~	opic
Chapter 9: Distributed Forces: Moments of Iner 9.7: Moments of Inertia of Composite Areas		1 11/5105	Sec	Ch
9.7: Moments of Inertia of Composite Areas	ertia (Continued)			
(Formulas for moments of inertia of common geometric shapes can be found on page 485) 9.8: Product of Inertia $I_{xy} = \int xy \ dA = \int (x'+\bar{x})(y'+\bar{y})dA$ $= \int x' y' \ dA + \bar{y} \int x' \ dA + \bar{x} \int y' \ dA + \bar{x} \bar{y} \int dA$ $I_{xy} == \bar{I}_{x'y'} + \bar{x} \bar{y} A$ 9.9: Principal Axes and Principal Moments of Inertia (Formulas for principle axis and principle moments of inertia can be found on pages 498-500) 9.10: Mohr's Circle for Moments and Products of Inertia Moments of Inertia of Masses 9.11: Moment of Inertia of Masses 9.11: Moment of Inertia of Masses 9.11: Moment of Inertia of a Mass $I = \int r^2 dm I = k^2 m or k = \sqrt{\frac{I}{m}}$ $I_x = \int (y^2 + z^2) \ dm I_y = \int (z^2 + x^2) \ dm$ $I_z = \int (x^2 + y^2) \ dm$ Note: This Chapter involves substantial amount of calculus-	[integration] → 12 th (to be taught) [differentiation] → 12 th (to be taught) [four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (2A), or (M7N1) → 7 th (2A) [area] (M3M3) (M3M4) → 3 rd (2B) [square root] (M8N1) → 8 th (2A) [coordinate system] (M4G3) → 4 th (2B) [areas of geometric shapes: circle, triangle] (M5M1) → 5 th (2B) [geometric shapes: ellipse] (MA2G4) → 10 th (2F) → To be taught as a special math topic [three-dimensional bodies: thin rectangular plate, rectangular prism] (M5M4) → 5 th (2B) [three-dimensional bodies: slender rod, circular cylinder, cone] (M6M3) → 6 th (2B) [three-dimensional bodies: circular cone, sphere] (M2G2) → 2 nd (2B) [trigonometric functions] (MA2G2) → 10 th (2F) → To be taught as a special math topic [cross product] → To be taught as a special math topic [partial differentiation] → 12 th (to be taught) [gradient: "del"] → 12 th (to be taught) [Inear algebral (MA2A5) (MA2A7) (MA2A8)	[force] (S4P3) → 4 th (3A) or (S8P3) → 8 th (3C) [power] (SP3) → 9 th (3C)	PS	PS

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to S the T	sible ade Start Fopic
	Math	Physics	Sec	Ch
Chapter 9: Distributed Forces: Moments of In	nertia (Continued)			
Chapter 9: Distributed Forces: Moments of In 9.12: Parallel-Axis Theorem $x = x' + \bar{x}$ $y = y' + \bar{y}$ $z = z' + \bar{z}$ $I_x = \int (y^2 + z^2) dm$ $I_x = \bar{I}_{x'} + m(\bar{y}^2 + \bar{z}^2) = \int [(y' + \bar{y})^2 + (z' + \bar{z})^2] dm$ $= \int (y'^2 + z'^2) dm + 2\bar{y} \int y' dm + 2\bar{z} \int z' dm + (\bar{y}^2 + \bar{z}^2) \int dm$ $I_y = \bar{I}_{y'} + m(\bar{z}^2 + \bar{x}^2)$ $I_z = \bar{I}_{z'} + m(\bar{x}^2 + \bar{y}^2)$ $I = \bar{I} + md^2$ $k^2 = \bar{k}^2 + d^2$ 9.13: Moments of Inertia of Thin Plates $I_{AA',mass} = \int r^2 dm$ $dm = \rho t dA$ $I_{AA',area} = \rho t \int r^2 dA$ $I_{AA',mass} = \rho t I_{AA',area} = \rho t \int r^2 dA$ $I_{AA',mass} = \rho t I_{CC'} = I_{AA'} + I_{BB'}$. Rectangular Plate $I_{AA',mass} = \rho t I_{AA',area} = \rho t (\frac{1}{12}a^3b)$ $I_{BB',mass} = \rho t I_{BB',area} = \rho t (\frac{1}{12}ab^3)$ $I_{AA'} = \frac{1}{12}ma^2$ $I_{BB'} = \frac{1}{12}mb^2$ $I_{CC'} = I_{AA'} + I_{BB'} = \frac{1}{12}m(a^2 + b^2)$ Circular Plate $I_{AA',mass} = \rho t I_{AA',area} = \rho t (\frac{1}{4}\pi r^4)$ $I_{AA'} = I_{BB'} = \frac{1}{4}mr^2$	nertia (Continued) [integration] → 12 th (to be taught) [differentiation] → 12 th (to be taught) [four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (2A), or (M7N1) → 7 th (2A) [area] (M3M3) (M3M4) → 3 rd (2B) [square root] (M8N1) → 8 th (2A) [coordinate system] (M4G3) → 4 th (2B) [areas of geometric shapes: circle, triangle] (M5M1) → 5 th (2B) [geometric shapes: ellipse] (MA2G4) → 10 th (2F) → To be taught as a special math topic [three-dimensional bodies: thin rectangular plate, rectangular prism] (M5M4) → 5 th (2B) [three-dimensional bodies: slender rod, circular cylinder, cone] (M6M3) → 6 th (2B) [three-dimensional bodies: circular cone, sphere] (M2G2) → 2 nd (2B) [trigonometric functions] (MA2G2) → 10 th (2F) → To be taught as a special math topic [cross product] → To be taught as a special math topic [partial differentiation] → 12 th (to be taught) [gradient: "del"] → 12 th (to be taught) [linear algebra] (MA2A6) (MA2A7) (MA2A8) (MA2A9) → 10 th (2G) → To be taught as a special math topic	[force] (S4P3) → 4 th (3A) or (S8P3) → 8 th (3C) [power] (SP3) → 9 th (3C)	PS	PS
$I_{CC} = I_{AA'} + I_{BB'} = \frac{1}{12}mr^2$				

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to S the T	sible ade Start Fopic
	Math	Physics	Sec	Ch
Chapter 9: Distributed Forces: Moments of In	nertia (Continued)			
9.14: Determination of the Moment of Inertia of a Three- Dimensional Body by Integration (Formulas for mass moments of inertia of common geometric shapes can be found on page 517). 9.15: Moment of Inertia of a Body with Respect to an Arbitrary Axis through O. Mass Products of Inertia $I_{OL} = \int p^2 dm = \int \vec{\lambda} \times \vec{r} ^2 dm$ $= \int [(\lambda_x y - \lambda_y x)^2 + (\lambda_y z - \lambda_z y)^2 + (\lambda_z x - \lambda_x z)^2]$ $= \lambda_x^2 \int (y^2 + z^2) dm + \lambda_y^2 \int (z^2 + x^2) dm + \lambda_z^2 \int (x^2 + y^2) dm - 2\lambda_x \lambda_y \int xy \ dm - 2\lambda_y \lambda_z \int yz \ dm - 2\lambda_z \lambda_x \int zx \ dm$ $I_{xy} = \int xy \ dm I_{yz} = \int yz \ dm I_{zx} = \int zx \ dm$ $I_{oL} = I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_{yz} - 2I_{zx} \lambda_z \lambda_x$ 9.17: Ellipsoid of Inertia. Principal Axes of Inertia $(OQ)\lambda_x = x (OQ)\lambda_y = y (OQ)\lambda_z = z$ $I_x x^2 + I_y y^2 + I_z z^2 - 2I_{xy} xy - 2I_{yz} zx = 1$ $I_y x'^2 + I_y y'^2 + I_z z'^2 = 1$	[integration] → 12 th (to be taught) [differentiation] → 12 th (to be taught) [four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (2A), or (M7N1) → 7 th (2A) [area] (M3M3) (M3M4) → 3 rd (2B) [square root] (M8N1) → 8 th (2A) [coordinate system] (M4G3) → 4 th (2B) [areas of geometric shapes: circle, triangle] (M5M1) → 5 th (2B) [geometric shapes: ellipse] (MA2G4) → 10 th (2F) → To be taught as a special math topic [three-dimensional bodies: thin rectangular plate, rectangular prism] (M5M4) → 5 th (2B) [three-dimensional bodies: circular code, sphere] (M2G2) → 2 nd (1B) [three-dimensional bodies: circular cone, sphere] (M2G2) → 2 nd (1B) [trigonometric functions] (MA2G2) → 9 th (2F) [cross product] → To be taught as a special math topic [partial differentiation] → 12 th (to be taught) [gradient: "del"] → 12 th (to be taught) [Inear algebra] (M2A2A6) (MA2A7) (MA2A8) (MA2A9) → 10 th (2G) → To be taught as a special math topic	[force] (S4P3) → 4 th (3A) or (S8P3) → 8 th (3C) [power] (SP3) → 9 th (3C)	PS	PS
$I_{OL} = I_{x'}\lambda_{x'}^{2} + I_{y'}\lambda_{y'}^{2} + I_{z'}\lambda_{z'}^{2}$				

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		Pos Gr to S the T	sible ade tart Fopic
	Math	Physics	Sec	Ch
Chapter 9: Distributed Forces: Moments of In	nertia (Continued)			
9.18: Determination of the Principal Axes and Principal	[integration] $\rightarrow 12^{\text{th}}$ (to be taught)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	PS
Moments of Inertia of a Body of Arbitrary Shape	[differentiation] \rightarrow 12 th (to be taught)	$[power] (SP3) \rightarrow 9^{th} (3C)$		
$ \nabla f = (2K)\tilde{r} K = \text{constant} \tilde{r} = x\hat{i} + y\hat{j} + z\hat{k} f(x, y, z) = I_x x^2 + I_y y^2 + I_z z^2 - 2I_{xy} xy - 2I_{yz} yz - 2I_{zx} zx - 1 \begin{vmatrix} I_x - K & -I_{xy} & -I_{zx} \\ -I_{xy} & I_y - K & -I_{yz} \\ -I_{zx} & -I_{yz} & I_z - K \end{vmatrix} = 0 (More calculus- and linear algebra- based formulas can be found n pages 534-535) $	[four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (2A), or (M7N1) → 7 th (2A) [area] (M3M3) (M3M4) → 3 rd (2B) [square root] (M8N1) → 8 th (1A) [coordinate system] (M4G3) → 4 th (2B) [areas of geometric shapes: circle, triangle] (M5M1) → 5 th (2B) [geometric shapes: ellipse] (MA2G4) → 10 th (2F) → To be taught as a special math topic [three-dimensional bodies: thin rectangular plate, rectangular prism] (M5M4) → 5 th (2B) [three-dimensional bodies: slender rod, circular cylinder, cone] (M6M3) → 6 th (2B) [three-dimensional bodies: circular cone, sphere] (M2G2) → 2 nd (2B) [trigonometric functions] (MA2G2) → 10 th (2F) → To be taught as a special math topic [cross product] → To be taught as a special math topic [partial differentiation] → 12 th (to be taught) [gradient: "del"] → 12 th to be taught) [linear algebra] (MA2A6) (MA2A7) (MA2A8) (MA2A9) → 10 th (2G) → To be taught as a special math topic			

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		e) Possil Grad to Sta the To	
	Math	Physics	Sec	Ch
Chapter 10: Method of Virtual Work				
10.1: Introduction	[integration] $\rightarrow 12^{\text{th}}$ (to be taught) [differentiation] $\rightarrow 12^{\text{th}}$ (to be taught)	[force] $(S4P3) \rightarrow 4^{th} (3A)$ or $(S8P3) \rightarrow 8^{th} (3C)$ [work] $(S8P3) \rightarrow 8^{th} (3C)$	PS	PS
10.2: Work of a Force	[trigonometric functions] (MA2G2) $\rightarrow 10^{\text{th}}$ (2F)	[potential energy] (SP3) \rightarrow 9 th (3C)		
$dU = \vec{F} \bullet d\vec{x} dU = F ds \cos \alpha dU = M d\theta$	\rightarrow To be taught as a special math topic			
10.3: Principle of Virtual Work	\rightarrow To be taught as a special math topic			
$\delta U = \vec{F}_1 \bullet \delta \vec{r} + \vec{F}_2 \bullet \delta \vec{r} + \dots + \vec{F}_n \bullet \delta \vec{r}$	[coordinate system] (M4G3) \rightarrow 4 th (2B)			
$= \left(\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n\right) \bullet \delta \vec{r} \rightarrow \delta U = \vec{R} \bullet \delta \vec{r}$	[partial differentiation] $\rightarrow 12^{\text{th}}$ (to be taught)			
10.4: Applications of the Principle of Virtual Work $x_{B} = 2\ell \sin \theta$ $y_{c} = \ell \cos \theta$				
$\delta x_{\rm B} = 2\ell\cos\theta\delta\theta \delta y_{\rm C} = -\ell\sin\theta\delta\theta$				
$\delta U = \delta U_{Q} + \delta U_{P} = -Q \delta x_{B} - P \delta y_{C}$				
$= -2Q\ell\cos\theta\partial\theta + P\ell\sin\theta\partial\theta$				
$\delta U = 0 \rightarrow$				
$2Q\ell\cos\theta\delta\theta = P\ell\sin\theta\delta\theta \rightarrow Q = \frac{1}{2}P\tan\theta$				
$B_x = -\frac{1}{2}P\tan\theta$				
10.5: Real Machines. Mechanical Efficiency				
$\delta U = -Q\delta x_{B} - P\delta y_{C} - F\delta x_{B}$				
$= -2Q\ell\cos\theta\delta\theta + P\ell\sin\theta\delta\theta - \mu P\ell\cos\theta\delta\theta$				
$\delta U = 0 \rightarrow 2Q\ell\cos\theta \ \delta\theta = P\ell\sin\theta \ \delta\theta - \mu P\ell\cos\theta \ \delta\theta \rightarrow$				
$n = \frac{\text{output wok}}{1 + \frac{2Q\ell\cos\theta}{2}} = \frac{2Q\ell\cos\theta}{2} \frac{\partial\theta}{\partial\theta}$				
'' input work $P\ell\sin\theta\delta\theta$				
$\eta = \frac{2\left(\frac{1}{2}P(\tan\theta - \mu)\right)\ell\cos\theta\ \delta\theta}{P\ell\sin\theta\ \delta\theta} = \frac{P(\tan\theta - \mu)\ell\cos\theta\ \delta\theta}{P\ell\sin\theta\ \delta\theta} = 1 - \mu\cot\theta$				

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Complet	ion Grade (Georgia Performance Standard Code)	Pos	sible
	[Pro requisite Moth Skills/Science Princ	viplos] (CPS Code) 🔿 Crede (Table No.)	Gr to S	ade
		ipies] (015 Coue) > 01aue (1able 110.)	the 1	Горіс
	Math	Physics	Sec	Ch
Chapter 10: Method of Virtual Work (Continued)				
10.6: Work of a Force during a Finite Displacement	[integration] $\rightarrow 12^{\text{th}}$ (to be taught)	[force] (S4P3) \rightarrow 4 th (3A) or (S8P3) \rightarrow 8 th (3C)	PS	PS
$dU = \vec{F} \bullet d\vec{r} \rightarrow U_{1 \to 2} = \int_{A_1}^{A_2} \vec{F} \bullet d\vec{r}$	[differentiation] \rightarrow 12 th (to be taught) [trigonometric functions] (MA2G2) \rightarrow 10 th (2F)	[work] (S8P3) \rightarrow 8 th (3C) [potential energy] (SP3) \rightarrow 9 th (3C)		
$dU = F ds \cos \alpha \rightarrow U_{1 \rightarrow 2} = \int_{S_1}^{S_2} (F \cos \alpha) ds$	→ To be taught as a special math topic [dot product]			
$dU = Md\theta \rightarrow U_{1 \to 2} = \int_{\theta_1}^{\theta_2} M \ d\theta U_{1 \to 2} = M(\theta_2 - \theta_1)$	→ To be taught as a special math topic [coordinate system] (M4G3) → 4^{th} (2B)			
Work of a weight	[partial differentiation] $\rightarrow 12^{\text{th}}$ (to be taught)			
$dU = -W dy \rightarrow U_{1 \to 2} = -\int_{y_1}^{y_2} W dy U_{1 \to 2} = -W (y_2 - y_1) = -W$	Фу			
Work of the force exerted by a spring				
$F = kx \rightarrow dU = -F dx = -kx dx$				
$U_{1\to 2} = -\int_{x_1}^{x_2} kx dx = \frac{1}{2} kx_1^2 - \frac{1}{2} kx_2^2 U_{1\to 2} = -\frac{1}{2} (F_1 + F_2) \Delta x$				
10.7: Potential Energy				
$U_{1\to 2} = \left(V_g\right)_1 - \left(V_g\right)_2 \leftarrow V_g = Wy$				
$U_{1\to 2} = (V_e)_1 - (V_e)_2 \leftarrow V_e = \frac{1}{2}kx^2$				
$dU = -dV U_{1 \to 2} = V_1 - V_2$				

Engineering Subject: Statics				
Engineering Analytic Topics & Typical Formulas	Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.) Math		Poss Gr to S the T	sible ade Start Fopic
	Math	Physics	Sec	Ch
Chapter 10: Method of Virtual Work (Continued)	ued)			
10.8: Potential Energy and Equilibrium $\frac{dV}{d\theta} = 0 V_e = \frac{1}{2} k x_B^2 V_g = W y_C x_B = 2\ell \sin \theta y_C = \ell \cos \theta$ $V_e = \frac{1}{2} k (2\ell \sin \theta)^2 V_g = W(\ell \cos \theta)$ $V = V_e + V_g = 2k\ell^2 \sin^2 \theta + W\ell \cos \theta$ $\frac{dV}{d\theta} = 4k\ell^2 \sin \theta \cos \theta - W\ell \sin \theta = 0$ $\frac{dV}{d\theta} = \ell \sin \theta (4k\ell \cos \theta - W) = 0$ 10.9: Stability of Equilibrium $\frac{dV}{d\theta} = 0 \frac{d^2V}{d\theta^2} > 0 : \text{ stable equilibriu m}$ $\frac{dV}{d\theta} = 0 \frac{d^2V}{d\theta^2} < 0 : \text{ unstable equilibriu m}$ $\frac{\partial V}{\partial \theta_1} = \frac{\partial V}{\partial \theta_2} = 0 \left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2} \frac{\partial^2 V}{\partial \theta_2^2} < 0$ $\frac{\partial^2 V}{\partial \theta_1^2} > 0 \text{or} \frac{\partial^2 V}{\partial \theta_2^2} > 0$	<pre>[integration] → 12th (to be taught) [differentiation] → 12th (to be taught) [trigonometric functions] (MA2G2) → 10th (2F) → To be taught as a special math topic [dot product] → To be taught as a special math topic [coordinate system] (M4G3) → 4th (2B) [partial differentiation] → 12th (to be taught)</pre>	[force] (S4P3) → 4 th (2A) or (S8P3) → 8 th (3C) [work] (S8P3) → 8 th (3C) [potential energy] (SP3) → 9 th (3C)	PS	PS
1 <u>2</u>	TE END	1	1	1

Table 3A. Pre-Calculus Based Statics Topics That Possibly Could Be Taught at 9th Grade

Chapter/Section	Page Numbers	Number of Pages
Chapter 1: Introduction (pp. 1-13 \rightarrow 13 pages sub-total. 6 sections out of	<u>(6)</u>	·
1.1: What Is Mechanics?	1-13	13
1.2: Fundamental Concepts and Principles		
1.3: Systems of Units		
1.4: Conversion from One System of Units to Another		
1.5: Method of Problem Solution		
1.6: Numerical Accuracy		
Chapter 2: Statics of Particles (pp. 15-63 → 49 pages sub-total. 15 section	ons out of 15)	
2.1: Introduction	15-63	49
2.2: Force on a Particle. Resultant of Two Forces		
2.3: Vectors		
2.4: Addition of Vectors		
2.5: Resultant of Several Concurrent Forces		
2.6: Resolution of a Force into Components		
2.7: Rectangular Components of a Force. Unit Vector		
2.8: Addition of Forces by Summing x and y Components		
2.9: Equilibrium of a Particle		
2.10: Newton's First Law of Motion		
2.11: Problems Involving the Equilibrium of a Particle. Free-Body Diagrams		
2.12: Rectangular Components of a Force in Space		
2.13: Force Defined by Its Magnitude and Two Points on Its Line of Action		
2.14: Addition of Concurrent Forces in Space		
2.15: Equilibrium of a Particle in Space		
Chapter 3: Rigid Bodies - Equivalent Systems of Forces (pp. 74-145 →	72 pages sub-total.	21 sections out of 21)
3.1: Introduction	74-145	72
3.2: External and Internal Forces		
3.3: Principle of Transmissibility. Equivalent Forces		
3.4: Vector Product of Two Vectors		
3.5: Vector Products Expressed in Terms of Rectangular Components		
3.6: Moment of a Force about a Point		
3.7: Varignon's Theorem		
3.8: Rectangular Components of the Moment of a Force		
3.9: Scalar Product of Two Vectors		
3.10: Mixed Triple Product of Three Vectors		

Table 3A. Pre-Calculus Based Statics Topics That Possibly Could Be Taught at 9th Grade (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 3: Rigid Bodies - Equivalent Systems of Forces (Continued)		
3.11: Moment of a Force about a Given Axis	↑	\uparrow
3.12: Moment of a Couple		1
3.13: Equivalent Couples		
3.14: Addition of Couples		
3.15: Couples Can Be Represented by Vectors		
3.16: Resolution of a Given Force Into a Force at <i>O</i> and a Couple		
3.17: Reduction of a System of Forces to One Force and One Couple		
3.18: Equivalent Systems of Forces		
3.19: Equipollent Systems of Vectors		
3.20: Further Reduction of a System of Forces		
3.21: Reduction of a System of Forces to a Wrench		
Chapter 4: Equilibrium of Rigid Bodies (pp. 158-210 → 53 pages sub-to	otal. 9 sections out of	of 9)
4.1: Introduction	158-210	53
4.2: Free-Body Diagram		
4.3: Reactions at Supports and Connections for a Two-Dimensional Structure		
4.4: Equilibrium of a Rigid Body in Two Dimensions		
4.5: Statically Indeterminate Reactions. Partial Constraints		
4.6: Equilibrium of a Two-Force Body		
4.7: Equilibrium of a Three-Force Body		
4.8: Equilibrium of a Rigid Body in Three Dimensions		
4.9: Reactions at Supports and Connections for a Three-Dimensional Structure		
Chapter 5: Distributed Forces: Centroids & Centers of Gravity		
(pp. 219-273 \rightarrow 55 pages sub-total. 0 sections out of 11)		
Chapter 6: Analysis of Structures (pp. 284-342 → 59 pages sub-total. 12	sections out of 12)	
6.1: Introduction	284-342	59
6.2: Definition of a Truss		
6.3: Simple Trusses		
6.4: Analysis of Trusses by the Method of Joints		
6.5: Joints under Special Loading Conditions		
6.6: Space Trusses		
6.7: Analysis of Trusses by the Method of Sections		
6.8: Trusses Made of Several Simple Trusses		
6.9: Structures Containing Multiforce Members		

Table 3A. Pre-Calculus Based Statics Topics That Possibly Could Be Taught at 9th Grade (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 6: Analysis of Structures (Continued)		
6.10: Analysis of a Frame	\uparrow	\uparrow
6.11: Frames Which Cease to Be Rigid When Detached from Their Supports	1	
6.12: Machines		
Chapter 7: Forces in Beams and Cables (pp. 353-401 \rightarrow 49 pages sub-total. 0 sections out of 10)		
Chapter 8: Friction (pp. 411-460 \rightarrow 50 pages sub-total. 8 sections out of	10)	
8.1: Introduction	411-441	31
8.2: The Laws of Dry Friction. Coefficients of Friction		
8.3: Angles of Friction		
8.4: Problems Involving Dry Friction		
8.5: Wedges		
8.6: Square-Threaded Screws		
8.7: Journal Bearings. Axle Friction		
8.9: Wheel Friction. Rolling Resistance	443-450	8
Chapter 9: Distributed Forces: Moments of Inertia (pp. 471-544 → 74 pages sub-total. 0 sections out of 18)		
Chapter 10: Method of Virtual Work (pp. 557-591→ 35 pages sub-total. 0 sections out of 9)		

Summary	
Total Number of Pages Covered by Text (Excluding "Review and Summary for 509	
Chapters," "Review Problems" and "Computer Problems Sections)	
Total Numbers of Sections Covered Under All Chapters 71 out of 121	
Percentage of Pre-Calculus Sections	
$\sim (\text{Number of Pre - Calculus Sections})_{(100\%)} - (71)_{(100\%)} - 58.7\%$	
$\frac{100\%}{\text{Pre-Calculus}} = \left(\frac{100\%}{121}\right)^{100\%} = \frac{100\%}{121} = \frac{10\%}{121} = \frac{10\%}{120\%} = $	
Total Numbers of Chapters Covered 6 out of 10	
Percentage of Chapters with Pre-Calculus Sections	
(Number of Chapters with Pre - Calculus Sections) (1000) (6) (1000) (6)	
$\%_{\text{Pre-Calculus}} = \left(\frac{100\%}{100\%}\right) = 00.0\%$	
Total Number of Pages Covered by Pre-Calculus Portion 285	
Percentage of Pre-Calculus Volume	
$($ Number of Pre - Calculus Pages $)_{(1000')} = (285)_{(1000')} = 56.00'$	
$\%_{\text{Pre-Calculus}} = \left(\frac{1}{\text{Total Number of Pages}}\right)(100\%) = \left(\frac{1}{509}\right)(100\%) = 36.0\%$	

Table 3A. Pre-Calculus Based Statics Topics That Possibly Could Be Taught at 9th Grade (Continued)

Table 3B. Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portion of Statics Topics to 9th Grade Students

Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code)		
[Pre-requisite Math Skills/Science Principles] (GPS Code) -> Grade (Table No.)		
Math	Physics	
Math 1. [areas of geometric shapes: circle, triangle, etc.] (M5M1) → 5 th and (2B) (M6M2) → 6 th (2C) 2. [coordinate system] (M4G3) → 4 th (2B) 3. [cross product] → To be taught as a special math topic 4. [dot product] → To be taught as a special math topic 5. [four operations] (M1N3) → 1 st (2A) + (M2N3) → 2 nd (1A), or (M7N1) → 7 th (2A) 6. [geometry: point, axis/line, 3D body] (M6G1) (M6G2) (M6M3) → 6 th (2B) 7. [linear algebra](MA2A6) (MA2A7) (MA2A8) (MA2A9) → 10 th (2G) → To be taught as a special math topic 8. [measurement: time] (M2M2) → 2 nd (2C) 9. [Parallelogram Law for the Addition of Force/Vector Graphics] (MA3A10) → 11 th (2H) → To be taught as a special math topic	Physics1. [acceleration] (S8P3) \rightarrow 8th (3C)2. [force] (S4P3) \rightarrow 4th (3A) or (S8P3) \rightarrow 8th (3C)3. [lever] (S4P3) \rightarrow 4th (3A)4. [motion] (SKP2) \rightarrow K (3A)5. [Newton's 1st, 2nd and 3rd Laws] (SP1) \rightarrow 9th (3C)6. [Newton's Law of Gravitation] (S8P5) \rightarrow 8th (3C)7. [scientific inquiry] (S7CS9) \rightarrow 7th (3B)	
10. [percent] (M5N5) \rightarrow 5 th (2A) 11. [problem-solving] (M3N5) \rightarrow 3 rd (2A) 12. [sigma pattice] (M6N1) \rightarrow 6 th (1A) or (MA1A2) \rightarrow 0 th (2E) \rightarrow To be taught as a		
 12. [sigma notation] (M6N1) → 6th (1A) or (MA1A3) → 9th (2E) → To be taught as a special math topic 13. [square root] (M8N1) → 8th (2A) 14. [surface] (M6M4) → 6th (2B) 15. [trigonometric functions] (MA2G2) → 10th (2F) → To be taught as a special math topic [unit conversion] (M6M1) → 6th (2C) 16. [unit conversion] (M6M1) → 0th (2U) → To be taught as a special math topic [unit conversion] (M6M1) → 0th (2U) 		
16. [vector graphics] (MA3A10) \rightarrow 9 th (2H) \rightarrow To be taught as a special math topic		

Table 4A. Calculus Based Statics Topics for Post-Secondary Engineering Education

Chapter/Section	Page Nos.	Chapter/Section	Page Nos.
Chapter 5: Distributed Forces: Centroids & Center	s of Gravity	Chapter 7: Forces in Beams and Cables	
5.1: Introduction	219-273	7.1: Introduction	354-401
5.2: Center of Gravity of a Two-Dimensional Body		7.2: Internal Forces in Members	
5.3: Centroids of Areas and Lines		7.3: Various Types of Loading and Support	
5.4: First Moments of Areas and Lines		7.4: Shear and Bending Moment in a Beam	
5.5: Composite Plates and Wires		7.5: Shear and Bending-Moment Diagrams	
5.6: Determination of Centroids by Integration		7.6: Relations among Load, Shear, and Bending Moment	
5.7: Theorems of Pappus-Guldinus		7.7: Cables with Concentrated Loads	
5.8: Distributed Loads on Beams		7.8: Cables with Distributed Loads	
5.9: Forces on Submerged Surfaces		7.9: Parabolic Cable	
5.10: Center of Gravity of a Three- Dimensional Body. Centroid of a		7.10: Catenary	
Volume		Chapter 8: Friction	
5.11: Composite Bodies		8.8: Thrust Bearings. Disk Friction	442-443
5.12: Determination of Centroids of Volumes by Integration		8.10: Belt Friction	450-460
Chapter 9: Distributed Forces: Moments of Inertia			
9.1: Introduction	472-544	9.10: Mohr's Circle for Moments and Products of Inertia	4
9.2: Second Moment, or Moment of Inertia, of an Area		9.11: Moment of Inertia of a Mass	`
9.3: Determination of the Moment of Inertia of an Area by Integration		9.12: Parallel-Axis Theorem	
9.4: Polar Moment of Inertia		9.13: Moments of Inertia of Thin Plates	
9.5: Radius of Gyration of an Area		9.14: Determination of the Moment of Inertia of a Three-Dimensional	
		Body by Integration	
9.6: Parallel-Axis Theorem		9.15: Moments of Inertia of Composite Bodies	
9.7: Moments of Inertia of Composite Areas		9.16: Moment of Inertia of a Body with Respect to an Arbitrary Axis	
		through O. Mass Products of Inertia	_
9.8: Product of Inertia		9.17: Ellipsoid of Inertia. Principal Axes of Inertia	_
9.9: Principal Axes and Principal Moments of Inertia		9.18: Determination of the Principal Axes and Principal Moments of	
		Inertia of a Body of Arbitrary Shape	
Chapter 10: Method of Virtual Work	-		
10.1: Introduction	557-591	10.6: Work of a Force during a Finite Displacement	_ ←
10.2: Work of a Force		10.7: Potential Energy	
10.3: Principle of Virtual Work		10.8: Potential Energy and Equilibrium	
10.4: Applications of the Principle of Virtual Work		10.9: Stability of Equilibrium	
10.5: Real Machines. Mechanical Efficiency			

Table 4B. Pre-Requisite Math and Science Topics to Be Reviewed Before Teaching the Calculus Portion of Statics Topics

Math & Science Pre-requisite Topics & Completion Grade (Georgia Performance Standard Code) [Pre-requisite Math Skills/Science Principles] (GPS Code) → Grade (Table No.)		
Math	Physics/Chemistry	
Math 1. [areas of geometric shapes: circle, triangle, etc.] (M3M3) (M3M4) \Rightarrow 3 rd (2B), (M5M1) \Rightarrow 5 th and (2B) (M6M2) \Rightarrow 6 th (2C) 2. [coordinate system] (M4G3) \Rightarrow 4 th (2B) 3. [cross product] \Rightarrow To be taught as a special math topic 4. [differentiation] \Rightarrow 12 th (to be taught) 5. [dot product] \Rightarrow To be taught as a special math topic 6. [four operations] (M1N3) \Rightarrow 1 st (2A) + (M2N3) \Rightarrow 2 ^{ul} (1A), or (M7N1) \Rightarrow 7 th (2A) 7. [geometry: point, axis/line, 3D body] (M6G1) (M6G2) (M6M3) \Rightarrow 6 th (2B) 8. [geometric shapes: ellipse] (MA2G4) \Rightarrow 10 th (2F) \Rightarrow To be taught as a special math topic 9. [gradient: "del"] \Rightarrow 12 th (to be taught) 10. [integration] \Rightarrow 12 th (to be taught) 11. [integration] \Rightarrow 12 th (to be taught) 12. [linear algebra](MA2A6) (MA2A7) (MA2A8) (MA2A9) \Rightarrow 10 th (2G) \Rightarrow To be taught as a special math topic 13. [logarithmic functions] (MA2A4) \Rightarrow 10 th (2E) 14. [measurement: area, weight, thickness] (M6M1) (M6M2) \Rightarrow 6 th (2C) 15. [measurement: ime] (M2M2) \Rightarrow 2 ^{sd} (2A) 16. [Parallelogram Law for the Addition of Force/Vector Graphics] (MA3A10) \Rightarrow 11 th (2H) \Rightarrow To be taught as special topic 17. [partial differentiation] \Rightarrow 12 th (to be taught) 18. [percent] (M5N5) \Rightarrow 5 th (2A) 20. [sigma notation] (M6N1) \Rightarrow 6 th (1A) or (MA1A3) \Rightarrow 9 th (2E) \Rightarrow To be taught as a special math topic 21. [special two-dimensional figures: parabolic spandrel, general spandrel] \Rightarrow To be taught as a special math topic 21. [square root] (M8N1) \Rightarrow 5 th (2A) 23. [surface] (M6M4) \Rightarrow 6 th (2B) 24. [three-dimensional bodies: thin rectangular plate, rectangular prism] (M5M4) \Rightarrow 5 th (2B) 25. [three-dimensional bodies: thin rectangular plate, rectangular prism] (M5M4) \Rightarrow 5 th (2B) 26. [three	 Physics/Chemistry 1. [acceleration] (S8P3) → 8th (3C) 2. [force] (S4P3) → 4th (3A) or (S8P3) → 8th (3C) 3. [lever] (S4P3) → 4th (3A) 4. [motion] (SKP2) → K (3A) 5. [Newton's 1st, 2nd and 3rd Laws] (SP1) → 9th (3C) 6. [Newton's Law of Gravitation] (S8P5) → 8th (3C) 7. [potential energy] (SP3) → 9th (3C) 8. [power] (SP3) → 9th (3C) 9. [scientific inquiry] (S7CS9) → 7th (3B) 10. [work] (S8P3) → 8th (3C) 	
33. [volume: ellipsoid, paraboloid] \rightarrow To be taught as a special math topic		

Conclusions and Recommendations

This report has presented (1) information about one popular college-level statics textbook (the first textbook or the "primary source of data) selected for the initial determination and selection of high school age-possible topics (Table 1), and (2) the outcomes of the research on the inclusion of mathematics, physics and chemistry concepts and skills needed for reading and homework assignments (Tables 2 through 4B). The following are recommended: (1) **Pilot study**: High schools could conduct pilot pedagogic experiments to determine the actual age-feasibility and age-appropriateness of all pre-calculus level statics-related analytic knowledge content identified in Table 3A (Pre-Calculus Based Statics Topics That Possibly Could Be Taught at 9th Grade), Table 3B (Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portion of Statics Topics to 9th Grade Students), using the selected textbook; and K-12 mathematics and science teachers could use the same Tables as references to incorporate statics topics into respective curriculum; and (2) **Curriculum development**: Existing K-12 engineering instructional materials or for the incorporation of statics-related knowledge and skills into their previously developed instructional materials.

References

- Committee on K-12 Engineering Education (2009). *Engineering in K-12 education: Understanding the status and improving the prospects*. Washington, DC: National Academy of Engineering and the National Research Council.
- Hacker, M. (2011). Private email correspondence, Saturday, January 22, 2011, 4:58:44 PM. Lewis, T. (2007). Engineering education in schools. *International Journal of Engineering Education*, 23(5), 843-852.
- Locke, E. (2009a). Proposed model for a streamlined, cohesive, and optimized k-12 STEM curriculum with a focus on engineering. *The Journal of Technology* Studies. Volume XXXV, Number 2, Winter 2009. Retrieved Thursday, February 17, 2011 from http://scholar.lib.vt.edu/ejournals/JOTS/v35/v35n2/pdf/locke.pdf.
- Locke, E. (2009b). *Report on the achievements of K-12 engineering education in Australia & its positive referential values for the evolution of a potentially viable K-12 engineering & technology curriculum in the United States*. Unpublished research document.

Smith, P. C., & Wicklein, R. C. (2007). *Identifying the essential aspects and related academic concepts of an engineering design curriculum in secondary technology education*. Unpublished internal research report, NCETE. Retrieved January 30, 2009 from http://ncete.org/flash/publications.php.

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